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(4) $r = a(1-\cos\theta)$

$$\frac{dr}{d\theta} = a\sin\theta$$

we have $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$

$$= \frac{1}{r^2} + \frac{1}{r^4}(a\sin\theta)^2$$

$$= \frac{1}{r^2} + \frac{a^2}{r^4}(\sin^2\theta)$$

$$\boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{a^2}{r^4}(1-\cos^2\theta)} \quad -(1)$$

Given $r = a(1-\cos\theta)$

$$\cancel{r = a(1-\cos\theta)}$$

$$\frac{r}{a} = (1-\cos\theta) \Rightarrow \cos\theta = 1 - \frac{r}{a} = \boxed{\frac{a-r}{a}} \quad -(2)$$

Substituting equation (2) in (1) we get

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{a^2}{r^4}\left[1 - \left(\frac{a-r}{a}\right)^2\right]$$

$$= \frac{1}{r^2} + \frac{a^2}{r^4}\left[\frac{a^2 - [a^2 + r^2 - 2ar]}{a^2}\right]$$

$$= \frac{1}{r^2} + \frac{\cancel{a^2}}{r^4}\left[\cancel{-a^2} - r^2 + 2ar\right]$$

$$= \frac{1}{r^2} + \frac{2ar}{r^4} - \frac{r^2}{r^4}$$

$$= \cancel{\frac{1}{r^2}} + \frac{2a}{r^3} - \cancel{1/r^2}$$

$$\frac{1}{p^2} = \frac{2a}{r^3}$$

$$\boxed{2ap^2 = r^3}$$